On the Synthesis of Equivalent Circuit Models for Multiports Characterized by Frequency-Dependent Parameters[1]

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Abstract - **The synthesis of lumped-element equivalent circuits for time-domain analysis of problems with frequency-dependent parameters is of great interest in microwave theory. This paper presents a systematic approach to generate minimal order realizations for passive microwave circuits characterized by either admittance, impedance or scattering parameter data. Also a very efficient method to ensure inherent system properties such as stability and passivity is described. Modeling examples for a two- and four-port system are given.**

I. INTRODUCTION

Efficient transient simulations of components characterized by measured or tabulated frequency-dependent data have been widely discussed in microwave literature. The frequency-dependent nature of the investigated problem generally requires: *i)* the definition of the simulation model in frequency-domain, *ii)* the transformation from the frequency- to the time-domain, and *iii)* the transient analysis and time-domain solution [1]. Commonly, the port response at discrete frequency points is availably in form of Y- (admittance), Z- (impedance) or S- (scattering) parameters resulting from broadband measurements or rigorous full-wave electromagnetic simulations. For the continuous representation in frequency-domain rational functions in the complex frequency $s = j\omega$ are used. From the rational functions the inverse Fourier transforms for closed-form representation in time-domain are computed analytically. The resulting simulation process in time-domain is resolved by using recursive convolution schemes. However, the numerical evaluation of convolution integrals can require strong computational effort and can cause numerical stability problems. Alternatively, macromodel synthesis techniques are applied to derive the corresponding equivalent circuit of the device under test (DUT). In that case the CPU-expenses for transient simulations are dependent on the order of the system model and the size of the synthesized lumped-element network, respectively.

This paper discusses different data fitting techniques used to generate a rational transfer-function (TF) representation for the discrete frequency response of multiports. A systematic approach is presented to determine the corresponding minimal order realization from the calculated TF-matrix and ensure inherent system properties such as stability and passivity in a very efficient manner. The models provide an accurate description of the investigated multiport within the specified frequency range and exhibit a considerably reduced complexity, which is verified by numerical examples.

II. DATA FITTING BY RATIONAL FUNCTIONS

It is assumed that the frequency responses of the n-port to be modeled are provided in terms of Y-, Z- or S-parameters at a discrete set of frequency points ω_i that covers the bandwidth of interest, as e.g. given in (1).

$$\mathbf{Y}(\omega_i) = \begin{pmatrix} Y_{11}(\omega_i) & \cdots & Y_{1n}(\omega_i) \\ \vdots & \ddots & \vdots \\ Y_{n1}(\omega_i) & \cdots & Y_{nn}(\omega_i) \end{pmatrix} \tag{1}$$

Y-, **Z**-, and **S**-matrices are connected to each other by equations (2)-(3), where $\mathbf{Z}_0$ is a diagonal matrix containing the square roots of all port impedances.

$$\mathbf{Y}(\omega_i) = \left(\mathbf{Z}_0^{-1} + \mathbf{S}(\omega_i)^{-1}\mathbf{Z}_0^{-1}\right)\left(\mathbf{Z}_0^{-1} - \mathbf{S}(\omega_i)\ \mathbf{Z}_0^{-1}\right) \tag{2}$$

$$\mathbf{Z}(\omega_i) = \left(\mathbf{Z}_0 - \mathbf{S}(\omega_i)^{-1}\mathbf{Z}_0\right)\left(\mathbf{Z}_0 + \mathbf{S}(\omega_i)\ \mathbf{Z}_0\right) \tag{3}$$

The rational TF-matrix that approximates the n-port frequency parameter data can be written as

$$\mathbf{H}(s) = \frac{\mathbf{A}_0 + \mathbf{A}_1 s + \mathbf{A}_2 s^2 + \ldots + \mathbf{A}_\varepsilon s^\varepsilon}{1 + b_1 s + b_2 s^2 + \ldots + b_\eta s^\eta}, \tag{4}$$

where b_0 is normalized to unity. The $\mathbf{A}_i$'s represent the

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n x n coefficient matrices of the numerator polynomials of the order ε; and the b_i's are the coefficients of the common denominator polynomial of the order η. $\mathbf{H}(s)$ can also be expressed in pole-residue form

$$\mathbf{H}(s) = \mathbf{K}_0 + \sum_{i=1}^{\eta} \frac{\mathbf{K}_i}{s - p_i}, \qquad (5)$$

where the p_i's are common poles and $\mathbf{K}_i$'s are residues of $\mathbf{H}(s)$. The coefficients of the TF-matrix (4) can be calculated using different complex curve fitting techniques [2].

The Model-Based-Parameter-Estimation method, e.g., represents the extension of Prony's approach to the treatment of frequency-domain data [1]. The $q = \eta + (\varepsilon + 1)\, n^2$ unknown coefficients in (4) are computed by applying a point-matching algorithm, which enforces the discrete data to be equal $\mathbf{H}(\omega_i)$ at $q/2$ frequency points ω_i. The resulting linear equation system for real and imaginary part of $\mathbf{H}(\omega_i)$ are straight forward to implement in computer code, however for high order approximations over a wide frequency range the system is highly ill conditioned. The problem can be overcome by using normalized angular frequency values $\omega_i^* = \omega_i / \omega_0$, splitting the frequency range in several sub-domains and/ or replacing the ordinary power series $\{1, \omega_i, \omega_i^2, \ldots, \omega_i^{q/2}\}$ in the linear equation system by orthogonal polynomials, such as Chebyshev polynomials [3].

A different method was developed by Gustavsen and Semlyen [4]. Their vector fitting procedure determines the unknown residue values in (5) in an iterative manner starting with an estimated set of real or complex conjugated pairs of poles p_i. The resulting over-determined equation system is solved by applying a least square based technique. A critical aspect of the vector fitting method is the choice of the appropriate starting poles.

III. Minimal Order Realization

Given a TF-matrix $\mathbf{H}(s)$, several forms of time-domain realizations can be obtained. The derivation of differential equations from a TF-system is referred as *macromodel synthesis*. In general, a set of first-order differential equations in state-space domain can be described as

$$\begin{aligned} \frac{d}{dt}\mathbf{x}(t) &= \mathbf{A}\cdot\mathbf{x}(t) + \mathbf{B}\cdot\mathbf{u}(t) \\ \mathbf{y}(t) &= \mathbf{C}\cdot\mathbf{x}(t) + \mathbf{D}\cdot\mathbf{u}(t) \end{aligned}, \qquad (6)$$

where $\mathbf{A} \in \mathbb{R}^{mxm}, \mathbf{B} \in \mathbb{R}^{mxn}, \mathbf{C} \in \mathbb{R}^{nxm}, \mathbf{D} \in \mathbb{R}^{nxn}$ and m equals the number of states, i.e. the order of the system. Using e.g. the Y-matrix (1), the k-th element of the input-vector $\mathbf{u}(t)$ and the output-vector $\mathbf{y}(t)$ equals the voltage $v_k(t)$ and the current $i_k(t)$ at port k, respectively.

A state-space realization (6) is said to be a minimal realization of the TF-matrix $\mathbf{H}(s)$ if the system matrix $\mathbf{A}$ in (6) has the smallest possible dimension, i.e. the fewest number of states m. The smallest dimension is called the *McMillan degree* of $\mathbf{H}(s)$ [5]. To calculate the minimal realization first either the left or right coprime factorization

$$\begin{aligned} \tilde{\mathbf{H}}(s) &= \mathbf{H}(s) - \mathbf{H}(s = \infty) \\ \tilde{\mathbf{H}}(s) &= \mathbf{D}_l(s)^{-1} \cdot \mathbf{N}_l(s) = \mathbf{N}_r(s) \cdot \mathbf{D}_r(s)^{-1} \end{aligned} \qquad (7)$$

is calculated. $\mathbf{D}_l, \mathbf{N}_l, \mathbf{D}_r, \mathbf{N}_r \in \mathbf{P}(s)^{nxn}$ are polynomial matrices in s. Considering the right coprime factorization, $\mathbf{D}_r$ and $\mathbf{N}_r$ can be decomposed in a higher order coefficient matrix $\mathbf{D}_{hc} \in \mathbb{R}^{nxn}$, where $det(\mathbf{D}_{hc}) \neq 0$, and lower order coefficient matrices $\mathbf{D}_{lc}, \mathbf{N}_{lc} \in \mathbb{R}^{nxm}$, respectively.

$$\begin{aligned} &\mathbf{D}_r(s) = \mathbf{D}_{hc}\mathbf{\Phi}(s) + \mathbf{D}_{lc}\mathbf{\Psi}(s) \\ &\mathbf{N}_r(s) = \mathbf{N}_{lc}\mathbf{\Psi}(s) \text{ with} \\ &\mathbf{\Phi}(s) = diag\left(s^{k_i}\right)\ \forall\ i = 1 \ldots n \end{aligned} \qquad (8)$$

From this the state-space system is obtained by

$$\begin{pmatrix} a_{11} & \cdots & a_{1m} \\ \vdots & \ddots & \vdots \\ a_{n1} & \cdots & a_{nm} \end{pmatrix} = -\mathbf{D}_{hc}^{-1}\mathbf{D}_{lc}, \quad \begin{pmatrix} b_{11} & \cdots & b_{1n} \\ \vdots & \ddots & \vdots \\ b_{n1} & \cdots & b_{nn} \end{pmatrix} = \mathbf{D}_{hc}^{-1} \qquad (9)$$

$$\mathbf{A} = \begin{pmatrix} \begin{pmatrix} 0 & 1 & 0 & \cdots & 0 \\ \vdots & & \ddots & \ddots & \vdots \\ a_{11} & & \cdots & & a_{1m} \end{pmatrix}_{k_1 x m} \\ \vdots \\ \begin{pmatrix} \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & 1 \\ a_{1n} & & \cdots & a_{nm} \end{pmatrix}_{k_n x m} \end{pmatrix} \quad \mathbf{B} = \begin{pmatrix} \begin{pmatrix} 0 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ b_{11} & \cdots & b_{1n} \end{pmatrix}_{k_1 x n} \\ \vdots \\ \begin{pmatrix} 0 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ b_{1n} & \cdots & b_{nn} \end{pmatrix}_{k_1 x n} \end{pmatrix} \qquad (10)$$

$$\mathbf{C} = \mathbf{N}_{lc} \qquad \mathbf{D} = \mathbf{H}(s = \infty).$$

In similar manner $\mathbf{A}$, $\mathbf{B}$, $\mathbf{C}$ and $\mathbf{D}$ can be derived from the left coprime factorization. The minimal realization (10) can be easily linked to standard nonlinear solvers or any general-purpose circuit simulator. Note that minimal realization implies minimal computational effort and less numerical stability problems. For those simulators, such as SPICE, that do not directly accept the differential equations as input, the state-space system can be converted to an equivalent circuit network consisting of passive elements and controlled voltage and current sources [6].

IV. System Properties

Essential to the usability of the synthesis process is that the generated model meets the system behavior of the DUT.

A. Stability

A critical aspect concerns the stability of the fitting model. It is assured that if all roots p_i of the common denominator polynomial in (4) lie in the left-hand side of the complex plane, i.e. $\mathrm{Re}\{p_k\} \leq 0$. In general stability can be enforced either as a constraint in the calculation of the rational approximation (4) or applying correction techniques, such as reflection/contraction of unstable poles to the left half-plane, in a very simple manner.

B. Passivity

More difficult to handle is to ensure passivity of the generated TF-representation. Passivity implies that the system cannot generate more energy than it absorbs, and no passive termination of the system will cause the system to become unstable. A passive system is asymptotically stable. However, asymptotic stability, i.e. $\mathrm{Re}\{p_k\} < 0$, does not imply passivity. The loss of passivity can be a serious problem because transient simulations of the generated system model in general circuit environment may encounter artificial oscillations.

$$\mathbf{I} - \mathbf{H}^T(j\omega)\cdot\mathbf{H}(-j\omega) \geq \mathbf{0} \qquad (10)$$

$$\mathbf{H}^T(-j\omega) + \mathbf{H}(j\omega) \geq \mathbf{0} \qquad (11)$$

Assuming the practical case $\mathbf{H}(s)$ being symmetric and its coefficients $\mathbf{A}_i$'s and b_i's being only real values, the network is passive if in case of S-matrix representation the matrix (10) or in case of Y- or Z-matrix representation the matrix (11) is positive definite for $0 \leq \omega \leq \infty$ [7]-[8]. However, ensuring that the condition (10) or (11), respectively, is not easy analytically for models expressed in form (4) or (5). On the other hand, transforming the S-matrix to the corresponding Y- or Z-matrix representation using (2)-(3) the *Kalman-Yakukubovich-Popov* criterion [9] can be applied. Using this criterion the resulting controllable and observable system representation (6) of the network is said to be passive, if there exist the matrices **L**, **W**, and **P**, with **P** being positive definite, satisfying the equation system (12).

$$\begin{aligned} \mathbf{PA} + \mathbf{A}^T\mathbf{P} &= -\mathbf{L}^T\mathbf{L} \\ \mathbf{PB} &= \mathbf{C}^T - \mathbf{L}^T\mathbf{W} \\ \mathbf{W}^T\mathbf{W} &= \mathbf{D}^T + \mathbf{D} \end{aligned} \qquad (12)$$

$$\begin{aligned} &\mathbf{P}(\mathbf{I}+\mathbf{A}) + (\mathbf{I}+\mathbf{A}^T)\mathbf{P} + \ldots \\ &\ldots(\mathbf{C} - \mathbf{PB})(\mathbf{D}^T+\mathbf{D})^{-1}(\mathbf{C} - \mathbf{B}^T\mathbf{P}) = \mathbf{0} \end{aligned} \qquad (13)$$

Ensuring this criterion results in only solving the *Ricatti-Equation* (13) and calculating the eigenvalues of the resulting matrix **P**. Now non-passive models can be detected at an early stage of the modeling process and with reduced computational effort.

V. Experimental Results

In order to demonstrate the usability of this advanced approach the following two examples are considered.

A. Coupled Flat Cable

First a coupled flat cable characterized by S-parameter data over the frequency range from 40 kHz to 130 MHz is considered. The fitting algorithm achieves excellent agreement between generated rational functions and dataset used with a TF-matrix of 12^{th}-order as plotted in Fig. 1. As described above the corresponding state-space system of *McMillan degree* 12 is calculated. The extracted equivalent circuit model for the 4-port is terminated with 50 Ohm at the ports 2, 3, and 4. At port 1 a 50-Ohm voltage source is exciting the system with a 5-V pulse of 70 ns duration. Unified transient simulations obtained using the calculated SPICE-compatible macromodel are compared with simulation results acquired with the original transmission-line model with frequency-dependent parameters in frequency-domain. As plotted in Fig. 2, excellent agreement is achieved

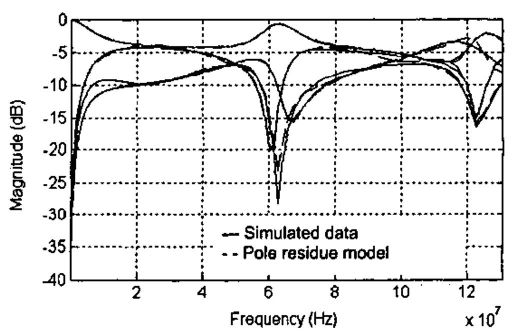

Fig. 1. Simulated S-parameters and calculated model of the investigated coupled flat cable.

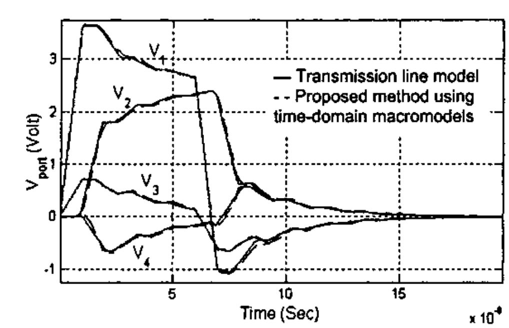

Fig. 2. Time-response of coupled flat cable obtained for a 5-V/70-ns pulse with a raise/fall-time of 10 ns.

B. Cable-Harness Antenna Coupling

As second example the coupling between a cable-harness and a antenna are investigated. A full-wave field solver is used to calculate the S-parameters from 150 kHz to 200 MHz. From this the Y-parameters are calculated and approximated by 22^{nd}-order pole-residue models. The magnitude responses of the data from simulation and pole-residue models are given in Fig. 3.

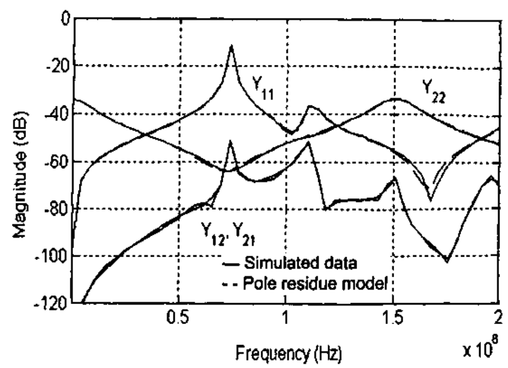

Fig. 3. Simulated Y-parameters and calculated model of the investigated harness antenna coupling application.

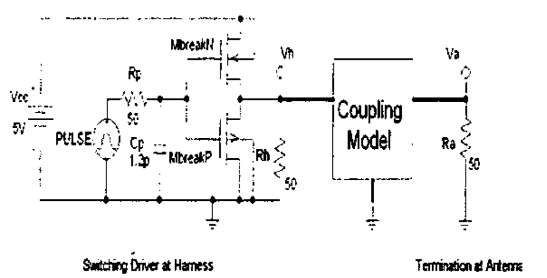

Fig. 4. Schematics of a harness antenna coupling with switching driver at the harness and 50-Ohm at the antenna feeding point.

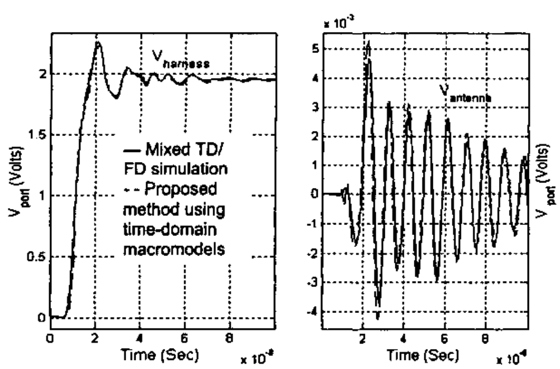

Fig. 5. Transient response at the harness (left) and the antenna feeding point (right) obtained with proposed method and mixed time/frequency domain simulations.

Fig. 4. illustrates the termination of the synthesized lumped element network. The transient response at the harness and antenna feeding point calculated with the proposed method compared with results obtained from mixed time- and frequency-domain simulation, shows excellent agreement, as depicted in Fig. 5.

VI. CONCLUSION

A systematic approach to extract a system model of minimal order from components characterized by frequency-dependent data has been described. The proposed method enables transient simulations in general circuit environment, consisting of lumped/distributed elements and nonlinear devices, with increased numerical stability, decreased model complexity and reduced computation times. This together with the ability to prove model properties in an efficient fashion makes this method very suitable for system modeling and time-domain analysis of frequency-dependent data.

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